Roll No. .

BCA–205(N)

B. C. A. (Second Semester)
EXAMINATION, May, 2018

(New Course)

Paper Fifth

MATHEMATICS–II

Time : Three Hours] [*Maximum Marks : 75*

Note : Section A is compulsory. Attempt any *seven* questions from Section B and any *one* question from Section C.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A 8 each

1. (a) If $u = e^{xyz}$, show that :

$$\frac{\partial^3 u}{\partial x\,\partial y\,\partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

(b) Show that the lines whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$ are at right angle.

2. (a) Discuss the maxima or minima of the function :

$$u = x^3 - y^3 - 3x$$

(b) Evaluate :

$$\int_0^1 \int_0^x e^{\frac{y}{x}}\,dx\,dy$$

Section—B 6 each

3. For any set A, B and C show that :

$$A - (B \cup C) = (A - B) \cap (A - C)$$

4. If R be a relation in the set of integer I defined by $R = \{(x, y) : x \in I, y \in I, x - y = 5k$ or $x - y$ is divisible by 5$\}$. Prove that R is an equivalence relation.

5. In a group of 25 people, 12 speak Hindi, 4 speak both English and Hindi and all people speak at least one of the two languages. How many people only English not Hind ? How many speak English ?

6. A plane meets the co-ordinate axes in A, B and C such that the controid of triangle ABC is the point (a, b, c). Show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$.

7. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, show that :

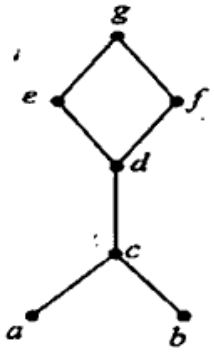
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

8. Evaluate the following triple integral :

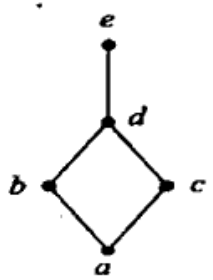
$$\int_{x=0}^{2} \int_{y=0}^{3} \int_{z=0}^{1} (x + y + z)\,dx\,dy\,dz$$

9. Determine whether the following Hasse diagrams represent lattice or not :

(i)



(ii)



10. Change the order of integration in the double integral $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dy\, dx$ and evaluate.

11. Find the equation to the sphere through the circle :

$$x^2 + y^2 + z^2 = 9$$

$$2x + 3y + 4z = 5$$

and the point (1, 2, 3).

12. Define distributive lattice and prove that in a distributive lattice, if an element has a complement, then this complement is unique.

Section—C 17 each

13. (a) Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9} and R be the relation on A × A defined as (a, b) R (c, d) if $a + d = b + c$. Prove that R is an equivalence relation.

(b) Prove that if f : A → B is one-one onto mapping, then f^{-1} : B → A will be one-one onto mapping.

14. (a) If :

$$z = e^u f(v)$$

$$u = ax + by$$

$$v = ax - by$$

show that :

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$$

(b) Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$.

15. (a) Prove that the lines :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

are coplanar, find their point of intersection. Also find the equation to the plane containing them.

(b) Find the area of between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.