

BCA-105(N)

B.C.A. (First Semester)

EXAMINATION, Dec.-2015

(New Course)

Paper- Fifth

MATHEMATICS-1

[Time: Three Hours]

[Maximum Marks: 75]

Note: Attempt questions from all the questions as directed.

SECTION-A

Note: Attempt all the questions, each will carry 3 marks. Short Answer Type Questions Only

1. A Show that the adjoint of

$$\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

is three times its transpose.

B. Find the equation of the line joining (1, 2) and (3, 6) using determinants.

$$\text{If } f(x) = \begin{cases} \frac{\sin [x]}{[x]}, & \text{if } [x] \neq 0 \\ 0, & \text{if } [x] = 0 \end{cases}$$

Where  $[x]$  denotes the greatest integer less than or equal to  $x$ , then find  $\lim_{x \rightarrow 0} f(x)$ .

Show that  $f(x) - [x]$  is differentiable at  $x = 0$ .

E. Differentiate  $\tan^{-1} \frac{x}{\sqrt{1-x^2}}$  with respect to  $\cos^{-1} (2x^2 - 1)$ .

F. Evaluate  $\int_0^{\infty} \frac{x^x (1-x)^x}{(1+x)^{2x}} dx$

G. Evaluate  $\int \frac{dx}{x(x^n+1)}$

H. Find the angle between the vectors  $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$  and  $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ .

I. If  $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$  and  $\vec{b} = 5\hat{i} + 5\hat{j} - \hat{k}$ , find  $(\vec{a} + \vec{b}) \times 2\vec{b}$  and  $(\vec{a} + \vec{b}) \times \vec{b}$

## SECTION-B

12 Marks Each Question

Long Answer Type Questions

(02 out of 04 questions)

Find the Eigen value and Eigen vector of the following matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

4. (a) Reduce the following matrix into normal form and hence find its rank

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(b) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

5. (a) Show that the following limit does not exist

$$\lim_{x \rightarrow 3} \frac{|x-3|}{x-3}$$

(b) Discuss the nature of discontinuity of the following function at the origin

$$f(x) = \begin{cases} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

## SECTION-C

12 Marks Each Question

Long Answer Type Questions

(02 out of 04 questions)

6. (a) Discuss the maxima and minima of the function

$$f(x) = \frac{x}{1+x^2}$$

(b) Trace the curve

$$Ay^2 = x^2 (a - x)$$

(a) Evaluate  $\int \sec^3 x dx$

(b) if  $I_n = \int_0^{\pi/4} \tan^n x dx$ ,

show that  $I_n + I_{n-2} = \frac{1}{n-1}$

8. (a) Evaluate  $\int_0^m x^n e^{-ax} dx$ , when  $m, n, a$  are positive constants.

(b) Evaluate  $\int \cos x \cos 2x \cos 3x dx$

9. (a) If  $(\vec{a}, \vec{b}, \vec{c})$  are three non-coplanar vectors show that  $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

(b) Prove that

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}.$$